Quantizer Design for Generalized Locally Optimum Detectors in Wireless Sensor Networks

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***Abstract*—We tackle distributed detection of a non-cooperative (i.e., whose emitted power is unknown) target with a wireless sensor network. When the target is present, sensors observe an (unknown) randomly fluctuating signal with attenuation depending on the distance between the sensor and the (unknown) target positions, embedded in Gaussian noise. The fusion center receives (only) sensor decisions through error-prone binary symmetric channels and is in charge of performing a more-accurate global inference. The resulting test is one-sided with nuisance parameters (i.e., the target position) present only under the hypothesis $\mathcal{H}_1$. To reduce the complexity of generalized likelihood ratio test, a generalized locally optimum detection test (based on Davies' framework) is investigated and a corresponding sensor threshold optimization (based on a semi-theoretical criterion) is developed and verified through simulations.**

***Index Terms*—Decentralized detection, threshold optimization, wireless sensor network, generalized likelihood ratio test, locally-optimum detection, data fusion.**

I. Introduction

Wireless Sensor Networks (WSNs) have drawn huge interest given their applicability to several fields, with distributed detection being one key task [1]. Due to strict bandwidth and energy constraints, each sensor usually sends one bit about the estimated hypothesis to the Fusion Center (FC). In this case, the optimal per-sensor test (under Bayesian/Neyman-Pearson frameworks) is a one-bit quantization of the local Likelihood-Ratio Test (LRT) [2], [3]. However, the search for quantization thresholds is exponentially complex and lack of knowledge of the parameters of the target to be detected precludes LRT sensor computation [4]. Thus the bit sent is either the result of a "dumb" quantization [5], [6] or embodies the estimated binary event, based on a sub-optimal rule [7].

In both cases, sensors bits are collected by the FC and fused via an intelligently-designed rule to improve individual detection rates; the optimum strategy, under conditional independence assumption, is a weighted sum, *with weights depending on unknown target parameters* [1]. Then, simple fusion approaches, based on the "counting rule" or simplified sensor assumptions, have been proposed to overcome such unavailability [8]–[11]. Indeed, the FC is faced to tackle a composite hypothesis test and the Generalized LRT (GLRT) is commonly employed [12]. To this end, GLRT-based fusion of quantized data was studied in [5], [13], and [14] for detecting: (*i*) a known source with unknown location, (*ii*) an unknown source with known observation coefficients, and (*iii*) an unknown source with unknown location. The latter case (i.e., the *uncooperative target* scenario) represents the most challenging, as it requires the *least knowledge* and one of the most interesting, given its applicability to surveillance [13], [14]. Also, to the best of our knowledge, only a few recent works have dealt with it. In [14], a GLRT was derived for this scenario and compared to the counting rule, the optimum rule and a "power-clairvoyant" GLRT, showing a marginal loss of GLRT with respect to the latter rule. Nevertheless, the GLRT requires a grid search on both the target location and emitted power domains, thus motivating the search for *simpler* fusion rules. Also, the test considered is one-sided with nuisance parameters present only under the alternative hypothesis, precluding the application of the usual Locally-Optimum Detection (LOD) test [12]. Hence, as a computationally simpler solution (i.e., not requiring a grid search for the target power), a Generalized LOD (G-LOD) test has been proposed in [15] and evaluated by simulations for a *fixed value* of local sensor thresholds.

In this letter, we tackle the decentralized detection of a non-cooperative target with a spatially-dependent *fluctuating* signature, with emitted power modeled as unknown and deterministic [15]. Specifically, each sensor observes a measurement on the target absence/presence, whose received signal is buried in additive Gaussian noise and experiences an Amplitude Attenuation Function (AAF) depending on the sensor-target distance. Each node forwards a sensor-optimal decision to a FC, over imperfect reporting channels (modelled as Binary Symmetric Channels, BSCs), being in charge of providing a more accurate global decision. The FC adopts the G-LOD test in [15] as an appealing alternative to GLRT and a *novel* threshold optimization is here proposed, based on a *semi–theoretical rationale* obtained from (asymptotic) Position-Clairvoyant (PC) LOD performance, not obtained in [15]. The resulting optimization is *per-sensor,* accounts for sensor-FC channel status, and requires *neither* the target power *nor* its position, so it can be computed *offline*. Simulation results are provided to compare the considered rules (and assess performance loss w.r.t. PC LOD) versus the local thresholds and verify the proposed design in a practical scenario.

This letter is organized as follows: Section II describes the system model; Section III recalls GLR and G-LOD tests, whereas in Section IV we focus on quantizer optimization; finally simulation results and concluding remarks are given in Section V.

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II. System Model

We consider a binary hypothesis test[1] where a collection of sensors $k \in \mathcal{K} \triangleq \{1, \ldots, K\}$ monitors a surveillance area to test the absence ($\mathcal{H}_0$) or presence ($\mathcal{H}_1$) of a target of interest having an isotropic, randomly-fluctuating, partially-specified spatial signature with distance-dependent path-loss:

$$\begin{cases} \mathcal{H}_0 : \; y_k = w_k \\ \mathcal{H}_1 : \; y_k = \xi_k \, g(\boldsymbol{x}_T, \boldsymbol{x}_k) + w_k \end{cases}, \tag{1}$$

where $y_k \in \mathbb{R}$ is kth sensor measurement, $w_k \sim \mathcal{N}(0, \sigma_{w,k}^2)$ denotes the measurement noise and the fading coefficient $\xi_k \sim \mathcal{N}(0, \theta)$ models fluctuations[2] in the received signal strength of the (same) target signature. The (common) average transmitted power θ is here assumed *unknown deterministic*, which well suits the case of a non-cooperative target. Also, $\boldsymbol{x}_T \in \mathbb{R}^d$ denotes the *unknown* target position, while $\boldsymbol{x}_k \in \mathbb{R}^d$ denotes the *known* kth sensor position, with the pair $(\boldsymbol{x}_T, \boldsymbol{x}_k)$ *uniquely* determining the value of $g(\boldsymbol{x}_T, \boldsymbol{x}_k)$, denoting the AAF. Then, $y_k \,|\, \mathcal{H}_0 \sim \mathcal{N}(0, \sigma_{w,k}^2)$ (resp. $y_k \,|\, \mathcal{H}_1 \sim \mathcal{N}(0, \, \theta \, g^2(\boldsymbol{x}_T, \boldsymbol{x}_k) + \sigma_{w,k}^2)$). As a corollary, henceforth, we will also discuss the case of $\boldsymbol{x}_T$ known, underlining similarities with results in [16].

We assume that sensors take their local decisions *individually*. Although each sensor is faced to tackle a *composite* hypothesis testing, such difficulty can be circumvented for this sensing model. To this end, we consider a local decision procedure based on the Neyman-Pearson lemma [12], i.e., a test based on the *local* Log Likelihood-Ratio (LLR) of kth sensor, i.e., $\lambda_k \triangleq \ln[\frac{p(y_k|\mathcal{H}_1)}{p(y_k|\mathcal{H}_0)}]$. Its explicit form $\lambda_k = \alpha_k + \beta_k \, y_k^2$, where $\alpha_k \triangleq \frac{1}{2} \ln\{\sigma_{w,k}^2 / [\sigma_{w,k}^2 + \theta \, g^2(\boldsymbol{x}_T, \boldsymbol{x}_k)]\}$ and $\beta_k \triangleq [\theta \, g^2(\boldsymbol{x}_T, \boldsymbol{x}_k)] / \{\sigma_{w,k}^2 [\sigma_{w,k}^2 + \theta \, g^2(\boldsymbol{x}_T, \boldsymbol{x}_k)]\}$ are both *positive*, reveals that the LLR is increasing with y_k^2, *irrespective* of θ and $\boldsymbol{x}_T$. Thus, by Karlin-Rubin theorem [17, p. 65], the *energy test* deciding in favour of $\mathcal{H}_1$ (resp. $\mathcal{H}_0$) when $y_k^2 > \gamma_k$ (resp. $y_k^2 \leq \gamma_k$), is *Uniformly Most Powerful* (UMP) in a local sense, where γ_k is a suitable threshold (chosen here to optimize the system performance, see Section IV). We assume that each sensor implements its *local* energy test, with performance summarized by the detection $P_{\mathrm{d},k} \triangleq \Pr\{y_k^2 \geq \gamma_k | \mathcal{H}_1\} = 2\, Q(\sqrt{\gamma_k / [\sigma_{w,k}^2 + \theta \, g^2(\boldsymbol{x}_T, \boldsymbol{x}_k)]})$ and false-alarm $P_{\mathrm{f},k} \triangleq \Pr\{y_k^2 \geq \gamma_k | \mathcal{H}_0\} = 2\, Q(\sqrt{\gamma_k / \sigma_{w,k}^2})$ probabilities [15], and sends the corresponding decision d_k (to cope with bandwidth/energy budgets in WSNs) to the FC.

Furthermore, to model energy-constrained reporting, we assume that kth sensor decision d_k is sent over a BSC and the FC observes an error-prone version due to non-ideal transmission, i.e., $\hat{d}_k = d_k$ (resp. $\hat{d}_k = (1 - d_k)$) with probability $(1 - P_{\mathrm{e},k})$ (resp. $P_{\mathrm{e},k}$), which we collect as $\widehat{\boldsymbol{d}} \triangleq \begin{bmatrix} \hat{d}_1 & \ldots & \hat{d}_K \end{bmatrix}^T$. Here $P_{\mathrm{e},k}$ denotes the (known) bit-error probability of kth link.

We notice that the *unknown* target position $\boldsymbol{x}_T$ is *observable* at FC *only* when the average power $\theta > \theta_0$ ($\theta_0 \triangleq 0$). Hence, the problem can be cast as *a one-sided test with nuisance parameters ($\boldsymbol{x}_T$) present only under the hypothesis $\mathcal{H}_1$* [18], where $\{\mathcal{H}_0, \mathcal{H}_1\}$ corresponds to $\{\theta = \theta_0, \theta > \theta_0\}$. The aim of this letter is the threshold-optimization of each sensor quantizer (i.e., an optimized γ_k, $k \in \mathcal{K}$) coupled with (computationally-simple) tests accepting $\mathcal{H}_1$ (resp. $\mathcal{H}_0$) when the generic statistic $\Lambda(\hat{\boldsymbol{d}})$ is above (resp. below) the threshold γ_{fc}. The FC performance is evaluated in terms of its false alarm ($P_{\mathrm{F}} \triangleq \Pr\{\Lambda > \gamma_{\mathrm{fc}} | \mathcal{H}_0\}$) and detection ($P_{\mathrm{D}} \triangleq \Pr\{\Lambda > \gamma_{\mathrm{fc}} | \mathcal{H}_1\}$) probabilities, respectively.

III. Fusion Rules

A common approach for composite hypothesis testing is given by the GLR [14]

$$\Lambda_{\mathrm{GLR}}(\hat{\boldsymbol{d}}) \triangleq 2 \, \ln[P(\hat{\boldsymbol{d}}; \hat{\theta}_1, \widehat{\boldsymbol{x}}_T) \, / \, P(\hat{\boldsymbol{d}}; \theta_0)], \tag{2}$$

where $(\hat{\theta}_1, \widehat{\boldsymbol{x}}_T)$ are the *Maximum Likelihood* (ML) *estimates* under $\mathcal{H}_1$ (i.e., $(\hat{\theta}_1, \widehat{\boldsymbol{x}}_T) \triangleq \arg\max_{(\theta, \boldsymbol{x}_T)} P(\hat{\boldsymbol{d}}; \theta, \boldsymbol{x}_T)$), with $\ln P(\hat{\boldsymbol{d}}; \theta, \boldsymbol{x}_T)$ denoting the log-likelihood function w.r.t. $(\theta, \boldsymbol{x}_T)$, whose explicit expression is [14], [15]

$$\sum_{k=1}^{K} \left\{ \widehat{d}_k \, \ln\left[\rho_{1,k}(\theta)\right] + (1 - \widehat{d}_k) \, \ln\left[1 - \rho_{1,k}(\theta)\right] \right\} \tag{3}$$

where $\rho_{1,k}(\theta, \boldsymbol{x}_T) \triangleq [(1 - P_{\mathrm{e},k}) \, P_{\mathrm{d},k}(\theta, \boldsymbol{x}_T) + P_{\mathrm{e},k}(1 - P_{\mathrm{d},k}(\theta, \boldsymbol{x}_T))]$ and a similar expression holds for $\ln P(\hat{\boldsymbol{d}}; \theta_0)$, when replacing $\rho_{1,k}$ with $\rho_{0,k} \triangleq [(1 - P_{\mathrm{e},k}) P_{\mathrm{f},k} + P_{\mathrm{e},k}(1 - P_{\mathrm{f},k})]$.

Eq. (2) reveals that Λ_{GLR} requires the solution to an optimization task, and the unavailability of the closed form of the ML estimate pair $(\hat{\theta}_1, \widehat{\boldsymbol{x}}_T)$ hinders its practical implementation. Hence, a grid approach on $(\theta, \boldsymbol{x}_T)$ is typically involved [13]–[15], with complexity $O(K \, N_{\boldsymbol{x}_T} \, N_\theta)$, where $N_{\boldsymbol{x}_T}$ (resp. N_θ) is the number of position (resp. power) bins employed.

A different path for exploiting the one-sided nature of the problem relies on Davies' work [18], which extends score-based tests to the case of nuisance parameters present only under $\mathcal{H}_1$.[3] If $\boldsymbol{x}_T$ were known, the LOD would represent a reasonable decision statistic for the corresponding one-sided testing [12]. However, since $\boldsymbol{x}_T$ is unknown in our setup, a *family of LOD statistics* is rather obtained by varying $\boldsymbol{x}_T$. Thus, to overcome this technical difficulty, Davies proposed the *family supremum* as the relevant statistic, that is:

$$\Lambda_{\mathrm{GLOD}}\left(\widehat{\boldsymbol{d}}\right) \triangleq \max_{\boldsymbol{x}_T} \left. \frac{\partial \ln\left[P\left(\widehat{\boldsymbol{d}}; \theta, \boldsymbol{x}_T\right)\right]}{\partial \theta} \right|_{\theta = \theta_0} \Big/ \sqrt{I(\theta_0, \boldsymbol{x}_T)}, \tag{4}$$

where $I(\theta, \boldsymbol{x}_T) \triangleq \mathbb{E}\{(\partial \ln[P(\widehat{\boldsymbol{d}}; \theta, \boldsymbol{x}_T)] / \partial\theta)^2\}$ denotes the Fisher Information (FI) assuming $\boldsymbol{x}_T$ *known*. Hereinafter, we will refer to the above decision test as *Generalized LOD* (G-LOD), to underline the use of LOD as the inner statistic within Davies framework [15]. The explicit expression of Λ_{GLOD} is

[1] *Notation* - Lower-case bold letters denote vectors, with a_n being the nth entry of $\boldsymbol{a}$; $\mathbb{E}\{\cdot\}$ and $(\cdot)^T$ denote expectation and transpose, respectively; $u(\cdot)$ denotes the Heaviside (unit) step function; $P(\cdot)$ and $p(\cdot)$ denote probability mass functions (pmf) and probability density functions (pdf), respectively; $\mathcal{N}(\mu, \sigma^2)$ denotes a Gaussian pdf with mean μ and variance σ^2; $Q(\cdot)$ (resp. $p_{\mathcal{N}}(\cdot)$) denotes the complementary cumulative distribution function (resp. the pdf) of a standard normal random variable, i.e., $\mathcal{N}(0, 1)$; the symbols $\sim$ and $\overset{a}{\sim}$ mean "distributed as" and "asymptotically distributed as".

[2] Due to spatial separation of the sensors, we assume that the noise contributions w_ks and the *fading* coefficients ξ_ks are both statistically independent.

[3] Indeed, score tests require the ML estimates of nuisances under $\mathcal{H}_0$ [12], which thus cannot be obtained, as they are not *observable* in our case.

drawn by using Eq. (3) and the FI closed form [15], given by $I(\theta, \boldsymbol{x}_T) = \sum_{k=1}^{K} \psi_k(\theta, \boldsymbol{x}_T)\, g^4(\boldsymbol{x}_T, \boldsymbol{x}_k)$, where

$$\psi_k(\theta, \boldsymbol{x}_T) \triangleq \frac{(1-2P_{\mathrm{e},k})^2\, \gamma_k}{P_{\mathrm{d},k}(\theta, \boldsymbol{x}_T)[1-P_{\mathrm{d},k}(\theta, \boldsymbol{x}_T)]} \times \frac{1}{(\theta g^2(\boldsymbol{x}_T, \boldsymbol{x}_k)+\sigma_{w,k}^2)^3} p_{\mathcal{N}}^2\left(\sqrt{\frac{\gamma_k}{\theta g^2(\boldsymbol{x}_T, \boldsymbol{x}_k)+\sigma_{w,k}^2}}\right). \quad (5)$$

The explicit form of G-LOD statistic has been obtained in [15] and shown to be $\Lambda_{\mathrm{GLOD}}(\widehat{\boldsymbol{d}}) \triangleq \max_{\boldsymbol{x}_T} \Lambda_{\mathrm{LOD}}(\widehat{\boldsymbol{d}}, \boldsymbol{x}_T)$, where

$$\Lambda_{\mathrm{LOD}}(\widehat{\boldsymbol{d}}, \boldsymbol{x}_T) = \frac{\sum_{k=1}^{K} \widehat{\nu}_k(\widehat{d}_k)\, g^2(\boldsymbol{x}_T, \boldsymbol{x}_k)}{\sqrt{\sum_{k=1}^{K} \psi_{k,0}\, g^4(\boldsymbol{x}_T, \boldsymbol{x}_k)}}, \quad (6)$$

denotes the LOD statistic assuming $\boldsymbol{x}_T$ known, and the auxiliary definitions $\widehat{\nu}_k(\widehat{d}_k) \triangleq (\widehat{d}_k - \rho_{0,k})\, \Xi_k$, $\psi_{k,0} \triangleq \rho_{0,k}(1-\rho_{0,k})\, \Xi_k^2$ and $\Xi_k \triangleq \frac{(1-2P_{\mathrm{e},k})\sqrt{\gamma_k}}{\rho_{0,k}(1-\rho_{0,k})\,(\sigma_{w,k}^2)^{3/2}}\, p_{\mathcal{N}}(\sqrt{\gamma_k/\sigma_{w,k}^2})$ have been employed. G-LOD appeal is motivated by its *simpler implementation* (since $\hat{\theta}_1$ is not needed), requiring solely a grid w.r.t. $\boldsymbol{x}_T$, that is $\Lambda_{\mathrm{GLOD}}(\widehat{\boldsymbol{d}}) \approx \max_{i=1,\ldots N_{\boldsymbol{x}_T}} \Lambda_{\mathrm{LOD}}(\widehat{\boldsymbol{d}}, \boldsymbol{x}_T[i])$. Thus, its complexity is $O(K\, N_{\boldsymbol{x}_T})$, implying a *significant complexity reduction* w.r.t. the GLR (e.g., $O(K\, N_{\boldsymbol{x}_T}\, N_\theta)$).

It is apparent that Λ_{GLOD} (as well as Λ_{GLR}, see Eqs. (2) and (6)) is a function of γ_k (through $\widehat{\nu}_k(\hat{d}_k)$ and $\psi_{k,0}$), $k \in \mathcal{K}$, (collected as $\boldsymbol{\gamma} \triangleq \begin{bmatrix}\gamma_1 & \cdots & \gamma_K\end{bmatrix}^T$) which can be *optimized* to boost performance. Section IV is devoted to fullfill this objective.

IV. Quantizer Design

We notice that (asymptotically-) optimal deterministic quantizers cannot be designed following [5], since no performance expressions are known in the literature for tests based on the Davies approach [18]. To this end, we adopt a *modified* version of the *rationale* in [5] and then we confirm its validity by simulations in Section V, as done in [6] for the non-fluctuating (two-sided) target case. Precisely, it is known that the (position $\boldsymbol{x}_T$) clairvoyant LOD statistic Λ_{LOD}, is asymptotically (and in weak-signal condition)[4] distributed as follows [12]:

$$\Lambda_{\mathrm{LOD}}(\boldsymbol{x}_T, \boldsymbol{\gamma}) \overset{a}{\sim} \begin{cases} \mathcal{N}(0,1) & \text{under } \mathcal{H}_0 \\ \mathcal{N}(\delta_Q(\boldsymbol{x}_T, \boldsymbol{\gamma}), 1) & \text{under } \mathcal{H}_1 \end{cases}, \quad (7)$$

where the deflection[5] $\delta_Q(\boldsymbol{x}_T, \boldsymbol{\gamma}) \triangleq (\theta_1 - \theta_0)\sqrt{\mathrm{I}(\theta_0, \boldsymbol{x}_T, \boldsymbol{\gamma})}$ (underlining dependence on $\boldsymbol{x}_T$ and $\boldsymbol{\gamma}$) is given as:

$$\delta_Q(\boldsymbol{x}_T, \boldsymbol{\gamma}) = \theta_1 \sqrt{\sum_{k=1}^{K} \psi_{k,0}(\gamma_k)\, g^4(\boldsymbol{x}_T, \boldsymbol{x}_k)}\,, \quad (8)$$

with θ_1 being the true value under $\mathcal{H}_1$. Clearly the larger $\delta_Q(\boldsymbol{x}_T, \boldsymbol{\gamma})$, the better the $\boldsymbol{x}_T-$clairvoyant LOD test will perform for the one-sided test assuming that the target to be detected is located at $\boldsymbol{x}_T$.[6] For this reason, we design the threshold vector $\boldsymbol{\gamma}$ so as to maximize $\delta_Q(\boldsymbol{x}_T, \boldsymbol{\gamma})$, that is $\boldsymbol{\gamma}^\star \triangleq \arg\max_{\boldsymbol{\gamma}}\ \delta_Q(\boldsymbol{x}_T, \boldsymbol{\gamma})$. In general, this procedure would lead to a $\boldsymbol{\gamma}^\star$ depending on $\boldsymbol{x}_T$ (and thus impractical). However, for this specific task, the optimization can be decoupled into the following set of K independent threshold design problems (then, the complexity of the threshold design scales *linearly* with K), which are also *independent of* $\boldsymbol{x}_T$ (see Eq. (8)):

$$\max_{\gamma_k} \left\{ \psi_{k,0}(\gamma_k) = \frac{1}{(\sigma_{w,k}^2)^2} \frac{p_{\mathcal{N}}^2\left(\sqrt{\gamma_k/\sigma_{w,k}^2}\right)(\gamma_k/\sigma_{w,k}^2)}{\Delta_k + P_{\mathrm{f},k}(\gamma_k)\left[1-P_{\mathrm{f},k}(\gamma_k)\right]} \right\}, \quad (9)$$

where $\Delta_k \triangleq [P_{\mathrm{e},k}(1-P_{\mathrm{e},k})]/(1-2P_{\mathrm{e},k})^2$. Such maximization is re-formulated in terms of the local false-alarm probability $P_{\mathrm{f},k}$ (being in *bijective* correspondence with γ_k), as

$$\kappa_k(P_{\mathrm{f},k}) = \frac{p_{\mathcal{N}}^2(\mathcal{Q}^{-1}(P_{\mathrm{f},k}/2))\left[\mathcal{Q}^{-1}(P_{\mathrm{f},k}/2)\right]^2}{\Delta_k + P_{\mathrm{f},k}\,(1-P_{\mathrm{f},k})}\,. \quad (10)$$

Examples of the objective $\kappa_k(P_{\mathrm{f},k})$ corresponding to different values of $P_{\mathrm{e},k}$ (viz. Δ_k) are shown[7] in Fig. 1. The optimal $P_{\mathrm{f},k}^\star$ can be easily evaluated via a 1-D numerical search.

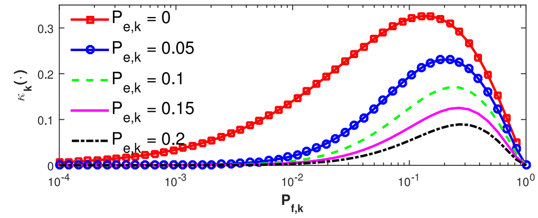

Fig. 1. Objective $\kappa_k(P_{\mathrm{f},k})$ for different $P_{\mathrm{e},k}$ values.

V. Simulation Results and Discussion

In this section we investigate threshold-optimization developed in Section IV by comparing GLR and G-LOD tests. To this end, we consider a 2-D area ($\boldsymbol{x}_T \in \mathbb{R}^2$) where a WSN with $K = 100$ sensors is employed to detect a target within the (square) surveillance region $\mathcal{A} \triangleq [0, 1]^2$. For simplicity, sensors are displaced on a regular square grid covering $\mathcal{A}$. For the sensing model, we assume $w_k \sim \mathcal{N}(0, \sigma_w^2)$, $k \in \mathcal{K}$ and, without loss of generality, we set $\sigma_w^2 = 1$. Also, the AAF chosen is $g(\boldsymbol{x}_T, \boldsymbol{x}_k) \triangleq 1/\sqrt{1+(\|\boldsymbol{x}_T - \boldsymbol{x}_k\|/\eta)^\alpha}$ (i.e., a power-law), where the target extent and decay exponent are set as $\eta = 0.2$ and $\alpha = 4$, respectively. Finally, we define the target Signal-To-Noise Ratio (SNR) as $\mathrm{SNR} \triangleq 10\log_{10}(\theta/\sigma_w^2)$.

According to Section III, Λ_{GLR} and Λ_{GLOD} are implemented by means of grids for θ and $\boldsymbol{x}_T$. Precisely, the search space of θ is assumed to be $S_\theta \triangleq [0, \bar{\theta}]$, where $\bar{\theta} > 0$ is such that SNR $= 20$ dB. The vector of grid points is then chosen as $\begin{bmatrix}0 & \boldsymbol{g}_\theta^T\end{bmatrix}^T$, where $\boldsymbol{g}_\theta$ collects power values corresponding to the SNR (dB) sampling $-10 : 1 : 20$ (thus $N_\theta = 32$). Differently, the search space of $\boldsymbol{x}_T$ is assumed to coincide with the surveillance area, i.e., $\mathrm{S}_{\boldsymbol{x}_T} = \mathcal{A}$, where the 2-D grid is obtained by regularly sampling it with $N_{\boldsymbol{x}_T} = N_c^2$ points, where $N_c = 100$. In this scenario, G-LOD requires the evaluation

[4] That is $|\theta_1 - \theta_0| = c/\sqrt{K}$ for some constant $c > 0$ [12].

[5] With a slight abuse of notation, we will employ the notation $\mathrm{I}(\theta, \boldsymbol{x}_T, \boldsymbol{\gamma})$ (resp. $\psi_{k,0}(\gamma_k)$) as opposed to $\mathrm{I}(\theta, \boldsymbol{x}_T)$ (resp. $\psi_{k,0}$) to underline their dependence on the local thresholds γ_k's.

[6] We recall that, in the case of one-sided testing, no tractable asymptotic expressions for GLRT are provided in the literature, see [12]. Nonetheless, aiming at a complete comparison, we will apply the proposed design also to GLRT in simulation results provided in Section V.

[7] Since the optimization is independent from $\boldsymbol{x}_T$, the objective $\kappa_k(P_{\mathrm{f},k})$ shares similarities with that provided in [16], pertaining to a *simpler* scenario.

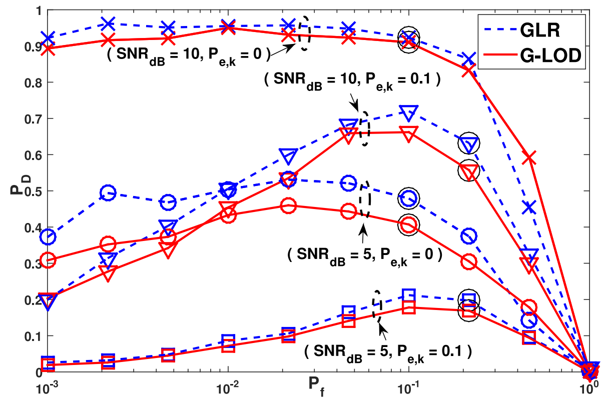

Fig. 2. P_{D} vs. $P_{\mathrm{f},k} = P_{\mathrm{f}}$, $P_{\mathrm{F}} = 0.01$; WSN with $K = 100$ sensors, $P_{\mathrm{e},k} \in \{0, 0.1\}$, SNR $\in \{5, 10\}$. Circled markers correspond to $P_{\mathrm{D}}(\boldsymbol{\gamma}^{\star})$.

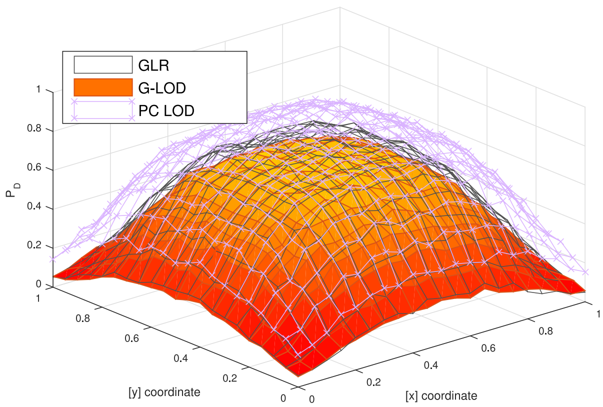

Fig. 3. P_{D} vs. $\boldsymbol{x}_T$, $P_{\mathrm{F}} = 0.01$; WSN with $K = 100$ sensors, $P_{\mathrm{e},k} = 0.1$, $\mathrm{SNR} = 10\,\mathrm{dB}$. $P_{\mathrm{f},k}$'s are chosen as the maximizers of Eq. (10).

of $N_c^2 = 10^4$ grid points, as opposed to $N_c^2 N_\theta = 3.2 \times 10^5$ points for GLR, i.e., a *reduction of complexity of roughly thirty times*.

First, in Fig. 2 we show P_{D} (under $P_{\mathrm{F}} = 0.01$, SNR $\in \{5, 10\}$ dB and $P_{\mathrm{e},k} \in \{0, 0.1\}$) vs. a common *sensor* (local) false-alarm probability $P_{\mathrm{f},k} = P_{\mathrm{f}}$, $k \in \mathcal{K}$ (imposed via a common threshold choice), for a target whose $\boldsymbol{x}_T$ is randomly drawn according to a uniform pdf within $\mathcal{A}$. It is apparent that the optimized threshold $\gamma_k^{\star}$ (viz. $P_{\mathrm{f},k}^{\star}$) proposed in Section IV represents an appealing solution (the corresponding P_{D} is highlighted with circled markers), since the optimal P_{D} in each curve depends on the target SNR experienced, which is *unknown*, and since a naively chosen P_{f} as in [15] may lead to significant performance degradation. This is especially true at low SNR, due to the assumptions required for drawing the asymptotic pdf in Eq. (7), and applies *also* to GLR.

Secondly, in Fig. 3, we report P_{D} (under $P_{\mathrm{F}} = 0.01$) vs. target location $\boldsymbol{x}_T$ (for $\mathrm{SNR} = 10\,\mathrm{dB}$, $P_{\mathrm{e},k} = 0.1$ and optimized $P_{\mathrm{f},k}^{\star}$), to obtain a clear comparison of detection rate over the surveillance area $\mathcal{A}$ and underline possibly *blind spots*. It is apparent that G-LOD test presents only *marginal loss* w.r.t. the GLRT, and a moderate loss with respect to a PC LOD test (Eq. (6), whose complexity is $O(K)$), which *unrealistically* assumes knowledge of $\boldsymbol{x}_T$ and thus represents an upper bound on achievable performance. Also the $P_{\mathrm{D}}(\boldsymbol{x}_T)$ profile is (qualitatively) similar for both rules, and underlines lower detection rate at the boundaries of the surveillance area, ascribable to *regularity* in displacement of the WSN within $\mathcal{A}$.

In summary, we studied performance and quantizer threshold design of a generalized version of the LOD test (based on [18]) for decentralized detection of a target emitting an unknown power (θ) at unknown location ($\boldsymbol{x}_T$), as a low-complexity alternative to GLRT (requiring a grid search over the space $(\theta, \boldsymbol{x}_T)$). We developed an appealing criterion (originated from semi-theoretical performance expressions) to optimize G-LOD sensor thresholds, resulting in *a simple per-sensor 1-D numerical search*. Such result was exploited to optimize the performance of G-LOD and GLR tests. Finally, it was shown via simulations that G-LOD test closely matches GLRT performance in the cases considered, while having moderate loss with respect to a PC LOD test. Future directions will account for fusion rules dealing with burstiness (time-correlation) of reporting channels.

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